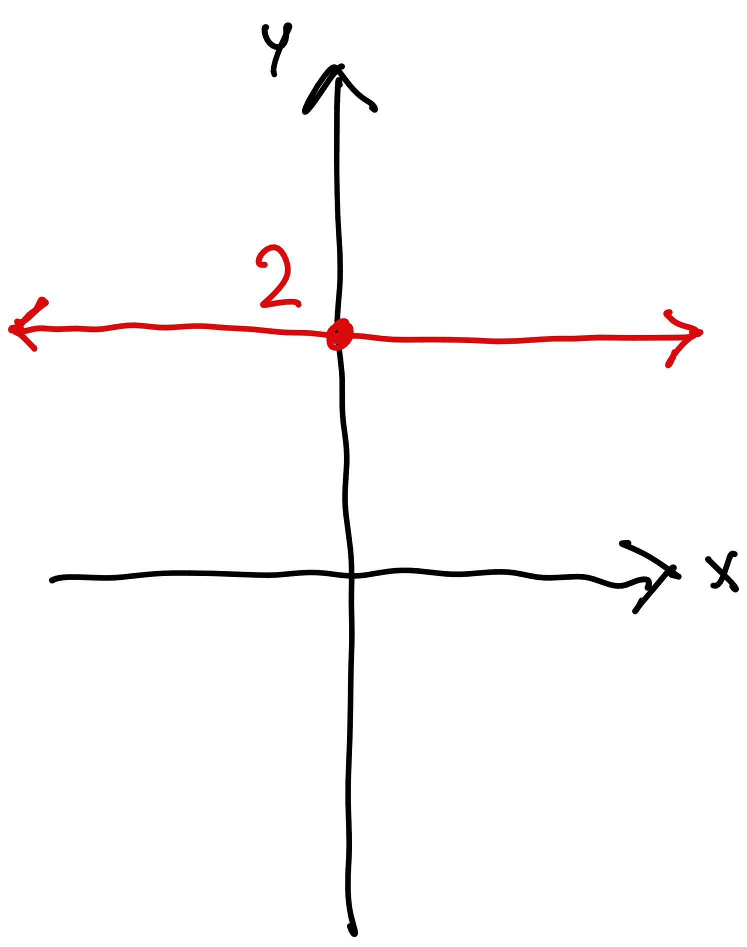
Class 15 - § 4.3 Graphs of polynomials

Review - Graph of basic functions

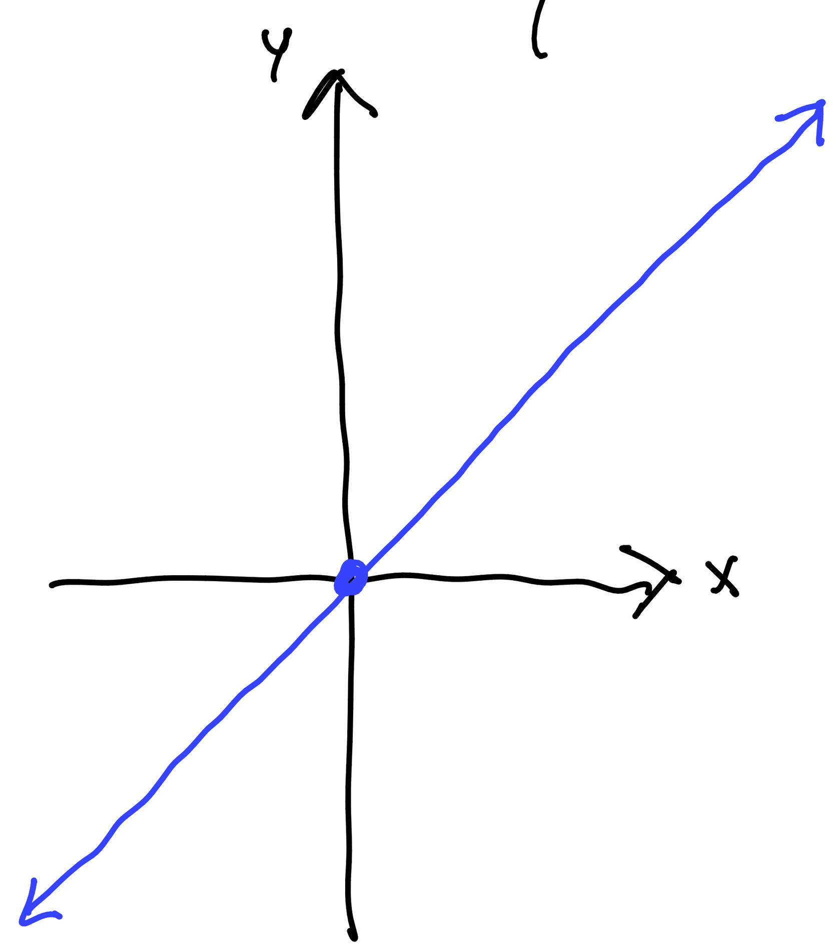
Constant



$f(x) = 2 = 2x^0$

polynomial of degree 0

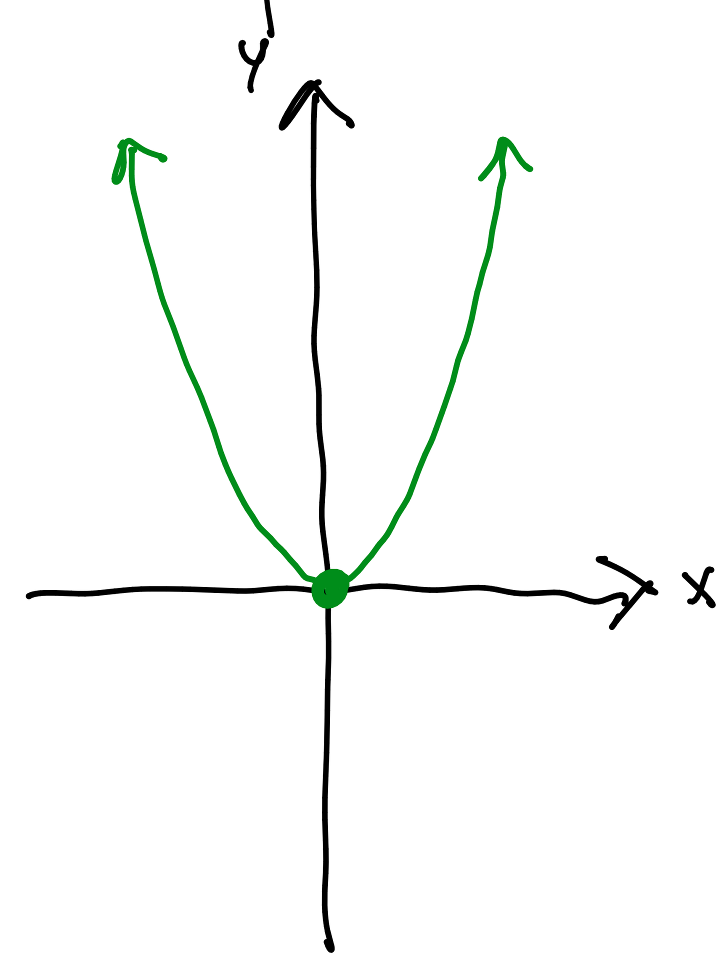
Identity



$f(x) = x = x^1$

poly. of degree 1

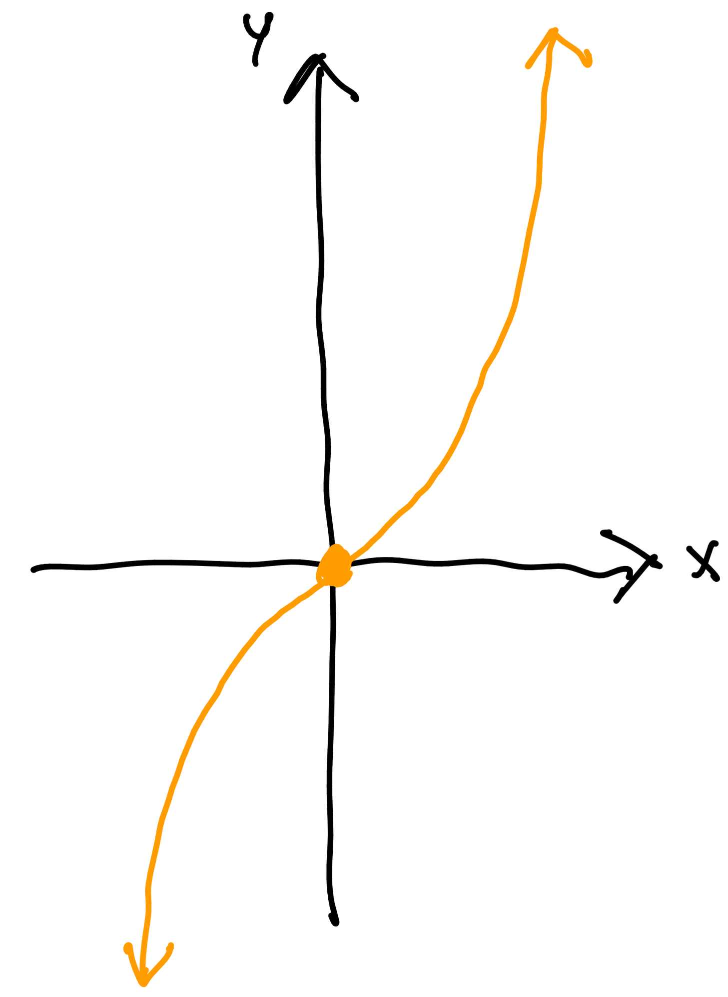
Square



$f(x) = x^2$

deg. 2

Cubic

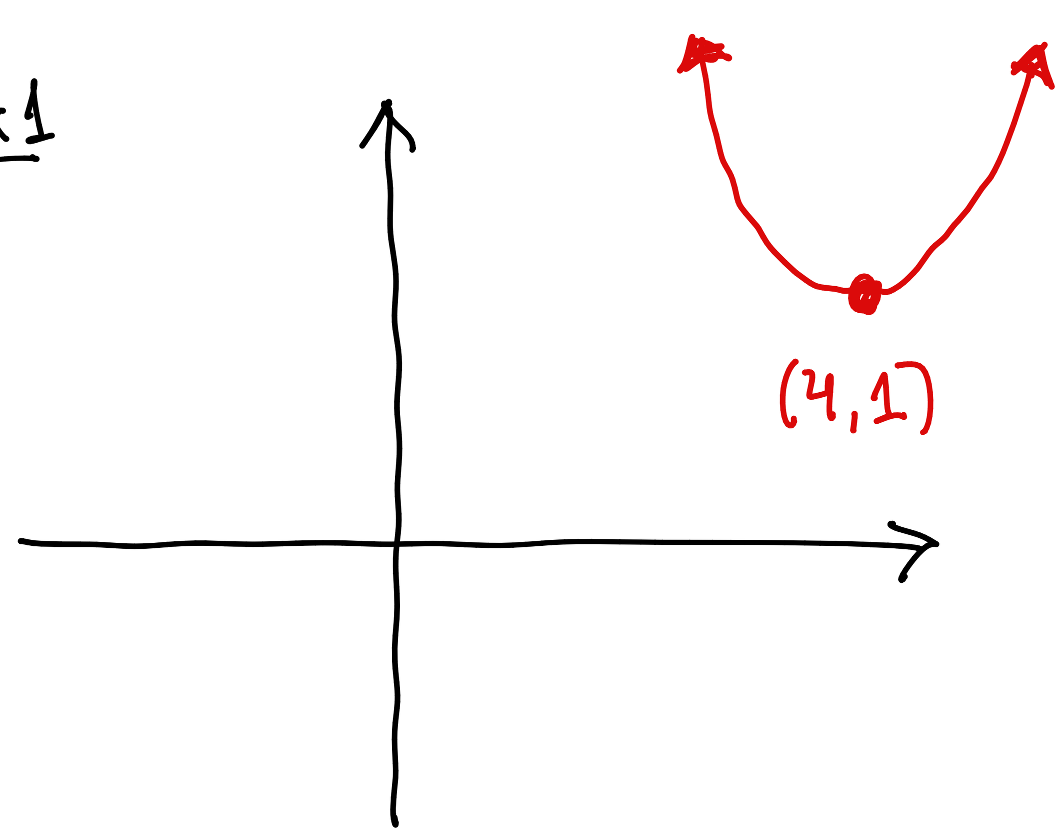


$f(x) = x^3$

deg. 3

→ Transformation of polynomial functions:

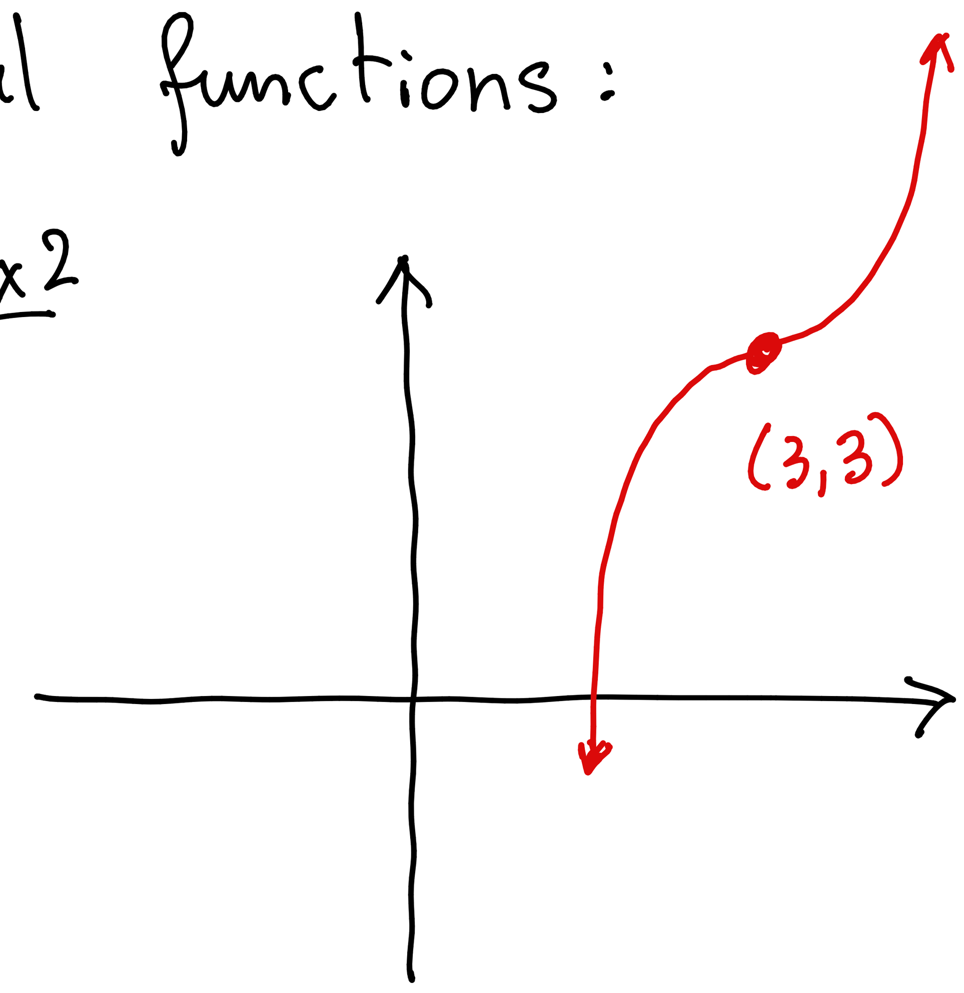
Ex1



basic function: x^2

$(x-4)^2 + 1$

Ex2



basic function: x^3

$(x-3)^3 + 3$

→ Evaluate polynomial functions

Ex: Let $f(x) = 4(x)(x-1)(x+2)$. Compute $f(0), f(2), f(-2)$.

$f(0) = 4\cdot(0)\cdot(-1)\cdot(+2) = 0$, $f(2) = 4\cdot(2)\cdot(1)\cdot(4) = 32$, $f(-2) = 4\cdot(-2)(-3)(0) = 0$

→ Identify polynomial functions

$\sqrt[n]{x^m} = x^{m/n}$

a) $f(x) = -3$

$f(x) = -3x^0$

poly. deg. 0

b) $g(t) = \dfrac{5t^2+7}{2}$

$g(t) = \frac{5}{2}t^2 + \frac{7}{2}t^0$

poly. deg. 2

c) $h(x) = \sqrt{\dfrac{x}{4}}$

$h(x) = \dfrac{\sqrt[2]{x^1}}{\sqrt{4}} = \frac{1}{2}x^{1/2}$

Root function

d) $p(x) = \dfrac{4}{5x^3-2}$

$p(x) = \dfrac{p_1(x)}{p_2(x)}$

Rational function

→ Solving polynomial equations

Ex 1: $-3(x-1)^2(x+1) = 0$

Ex 2: $-\frac{1}{3}(x^2-36)^2 = 0$

factor method

Ex 3: $\underbrace{9x^4}_{9\cdot 25} - 225x^2 = 0$

Ex 4: $\underbrace{x^3 - 7x^2}_{x^2} \underbrace{- 4x + 28}_{-4} = 0$ $\quad (-4)(-7)$

1st factor

factor method

a) $x = 1, -1$

b) $x = 6, -6$

$\boxed{\begin{array}{l} \text{c) } 9x^2(1x^2 - 25) = 0 \\ \text{d) } (1x^2-4)(1x-7) = 0 \end{array}}$

c) $x = 0, -5, 5$

$LC = 9 \cdot 1^1 = 9$

$\deg = 2 + 2 \cdot 1 = 4$

d) $x = 2, -2, 7$

$\deg = 2 \cdot 1 + 1 \cdot 1 = 3$

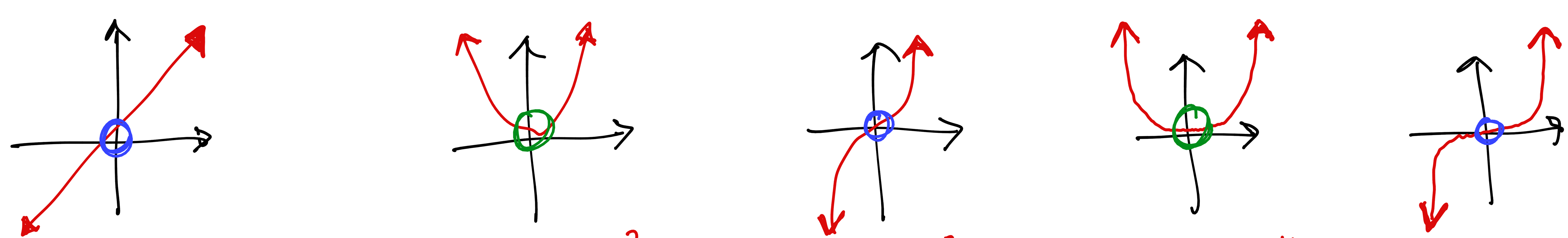
$LC = 1^1 \cdot 1^1 = 1$

§4.3 Graphs of Polynomials

Polynomials: $\dfrac{1}{\sqrt{2}}x^8 + \dfrac{4}{3}x^5 - 2\sqrt{2}\,x^4 - 1x^0$

Real numbers

non-negative exponents



$f(x) = x$ $\quad$ $f(x) = x^2$ $\quad$ $f(x) = x^3$ $\quad$ $f(x) = x^4$ $\quad$ $f(x) = x^5$

When we have odd exponent, the graph crosses the origin.

When we have even exponent, the graph bounces at the origin.

If odd exponent, the ends are opposite (up & down) ↙ ↗

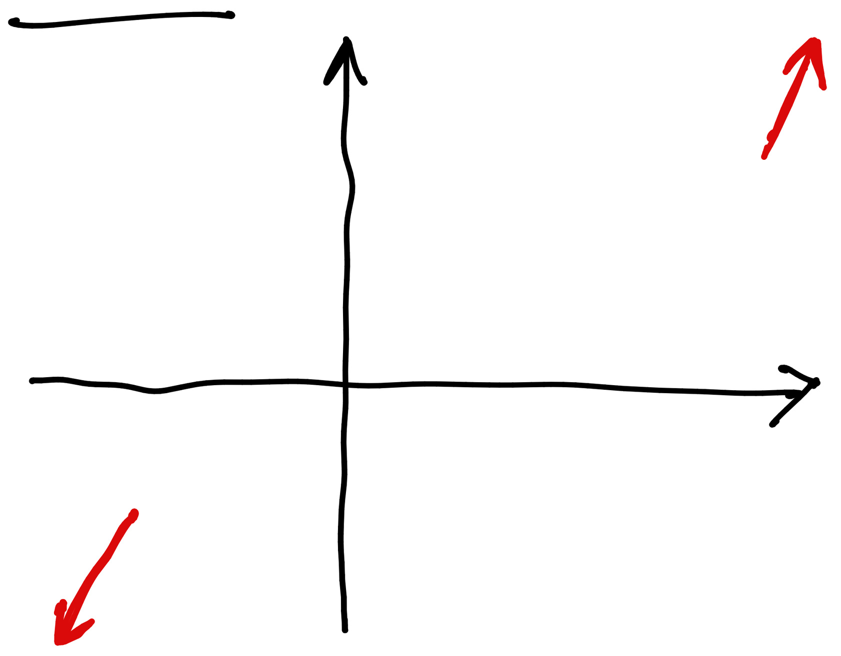
If even exponent, the ends are the same ↖ ↗

The end behavior of a polynomial function

<u>Def</u>: the number multiplying "x" raised to the largest exponent is called **leading coefficient.** (LC)

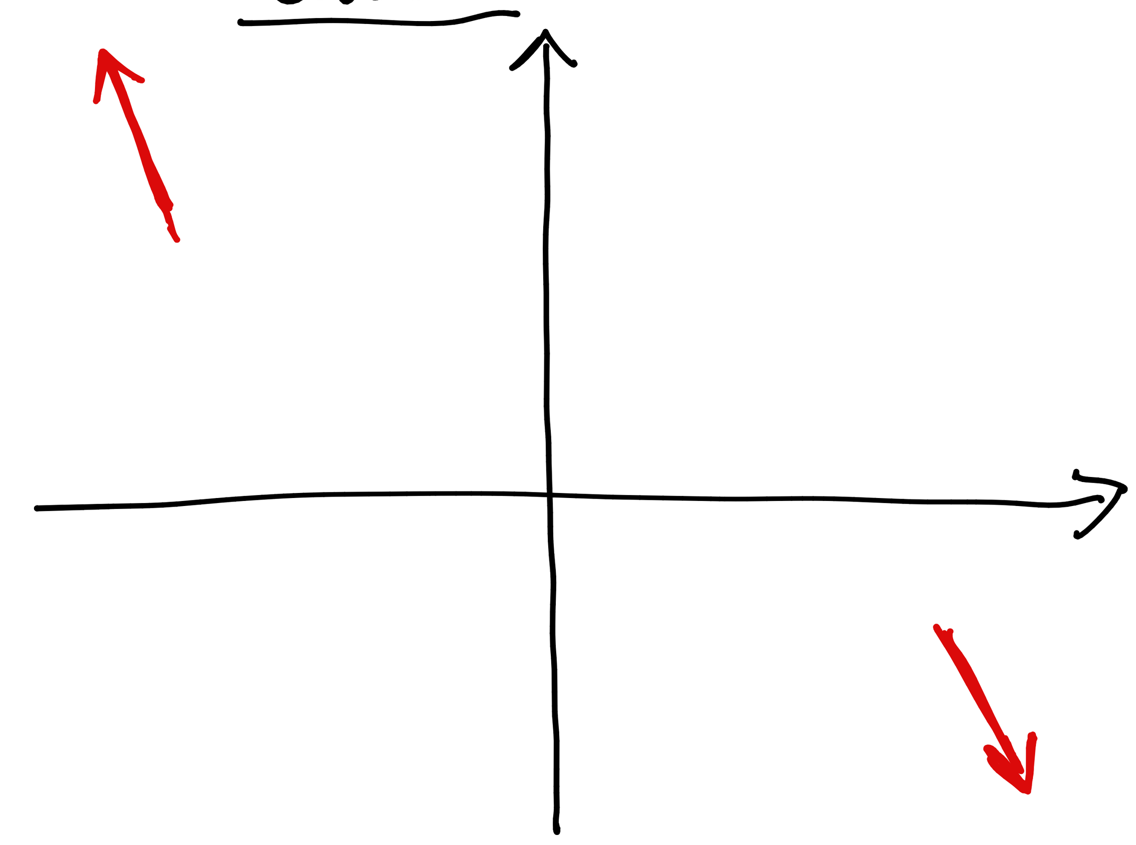
<u>Ex</u>: the polynomial $\frac{1}{\sqrt{2}}x^8 + \frac{4}{3}x^5 - 2\sqrt{2}x^4 - 1x^0$ has leading coefficient $\frac{1}{\sqrt{2}}$

<u>case 1</u>



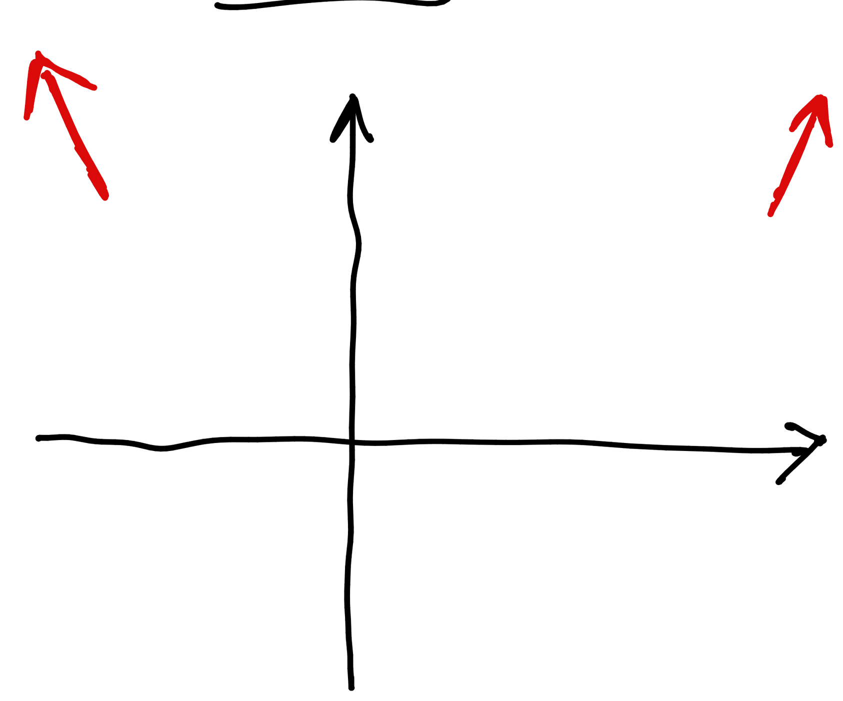
LC > 0 & odd deg

<u>case 2</u>



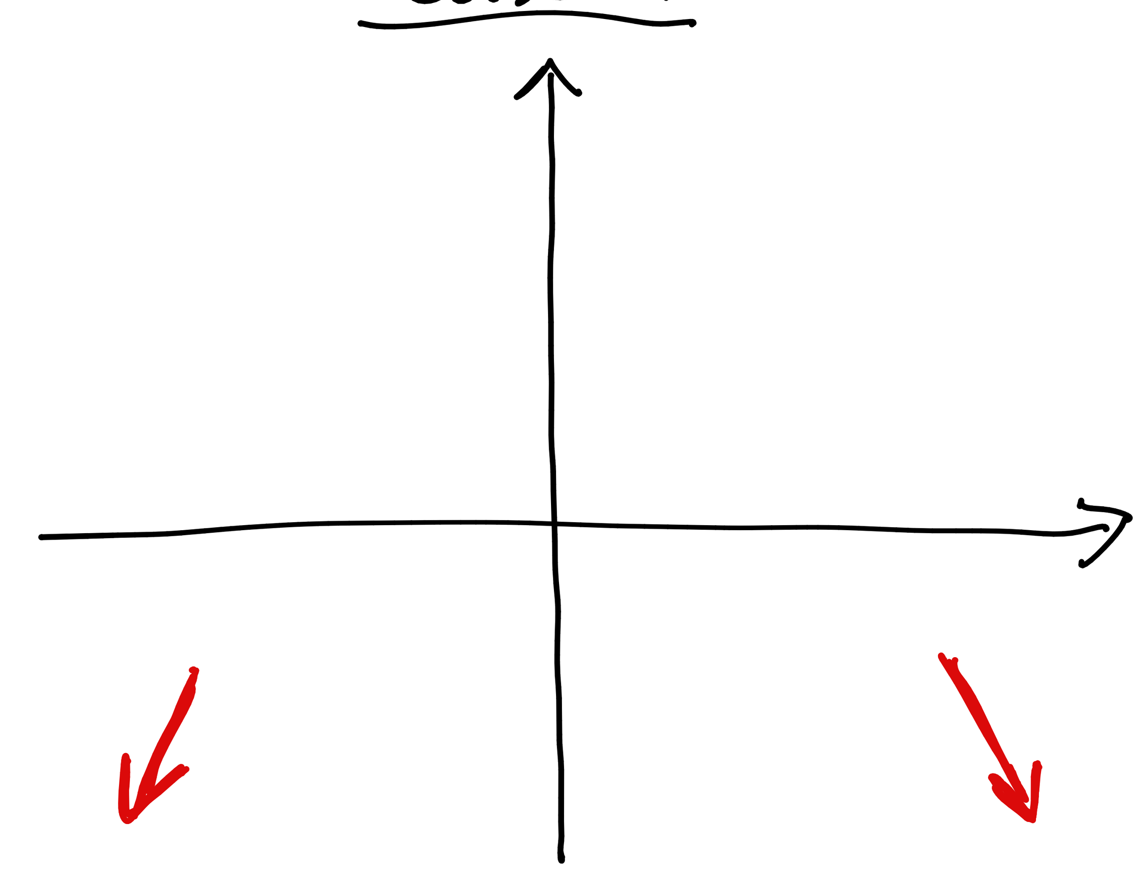
LC < 0 & odd deg.

<u>case 3</u>



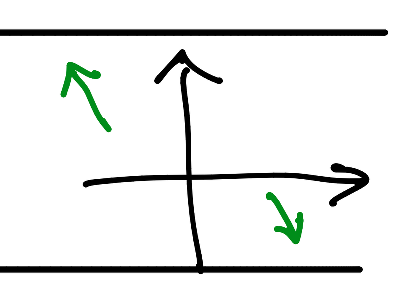
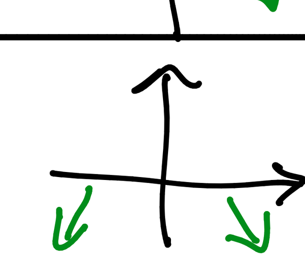
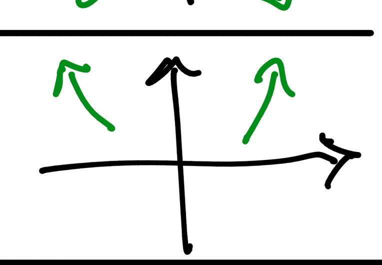
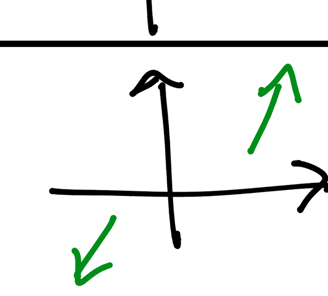
LC > 0 & even deg

<u>case 4</u>



LC < 0 & even deg.

<u>Exercise</u>: For each polynomial, determine <u>degree</u>, <u>LC</u> & <u>graph case</u>.

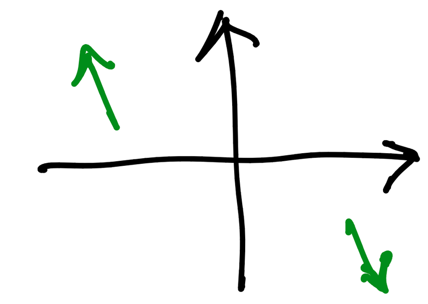
polynomial	degree	LC	case	graph
a) $-3\underbrace{(1x-1)^2}_{2}\underbrace{(1x+1)^1}_{1}=0$	$1+2=3$	$(-3)\cdot(1)^2\cdot(1)^1=-3$	2	
b) $-\frac{1}{3}\underbrace{(1x^2-36)^2}=0$	4	$-\frac{1}{3}\cdot(1)^2=-\frac{1}{3}$	4	
c) $9x^4-225x^2=0$	4	9	3	
d) $1x^3-7x^2-4x+28=0$	3	1	1	

e) $-7(1x^2-1)^{2}(1x^3-8)^{3}(2x^2-2)^{5}$. deg $= 2\cdot2+3\cdot3+2\cdot5$ $= 23$

$$(-7)^{1}\cdot(1x^{2})^{2}\cdot(1x^{3})^{3}\cdot(2x^{2})^{5} = -224x^{23}$$

LC $= (-7)^{1}\cdot1^{2}\cdot1^{3}\cdot2^{5}$
$= -7\cdot1\cdot1\cdot32$
$= -224$

case : 2



Def: the number "c" is a root or a zero of a polynomial if $f(c)=0$ for a polynomial function $f(x)$.

Def: the multiplicity of a factor is the power (exponent) of that factor.

Example: in the polynomial a) $f(x) = -3(x-1)^2(x+1)$ we have:

- 1 and -1 as roots or zeros because $f(1)=0$ & $f(-1)=0$;
- the factor $(x-1)^{2}$ has multiplicity 2 & $(x+1)^{1}$ has multiplicity 1;

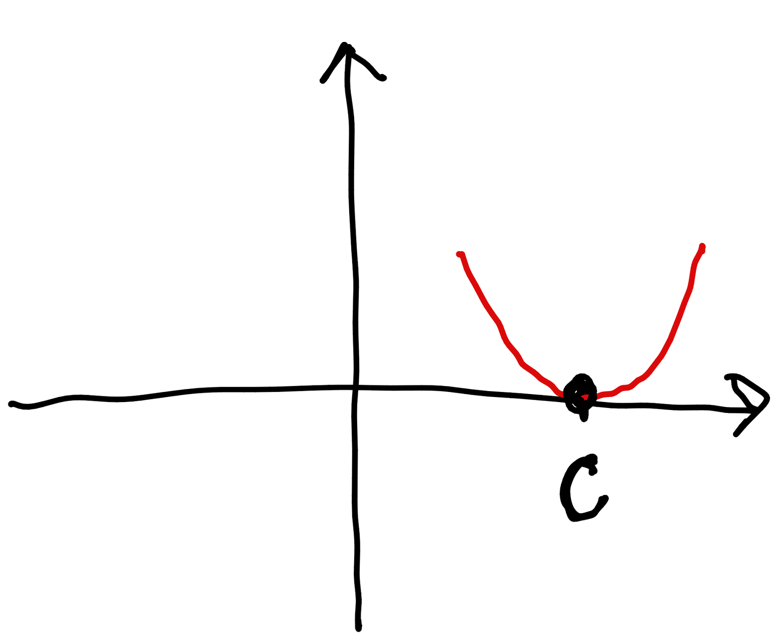
$(x-1)$: root / zero

$(x-(-1))^1$: root / zero

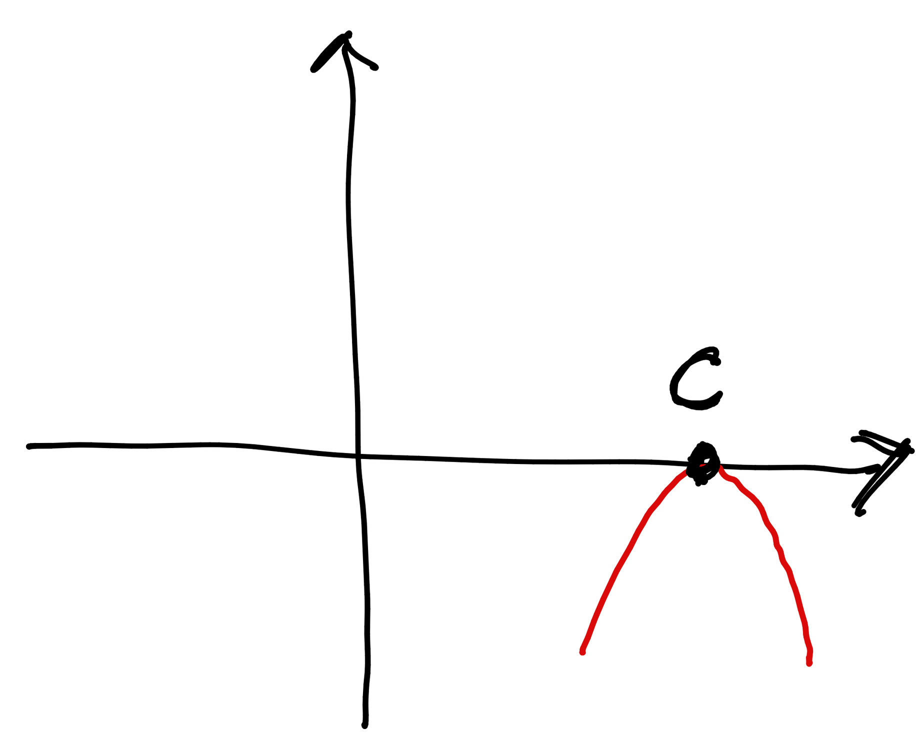
Determine the zeroes & multiplicities of polynomials

Consider the factor $(x-c)^k$:

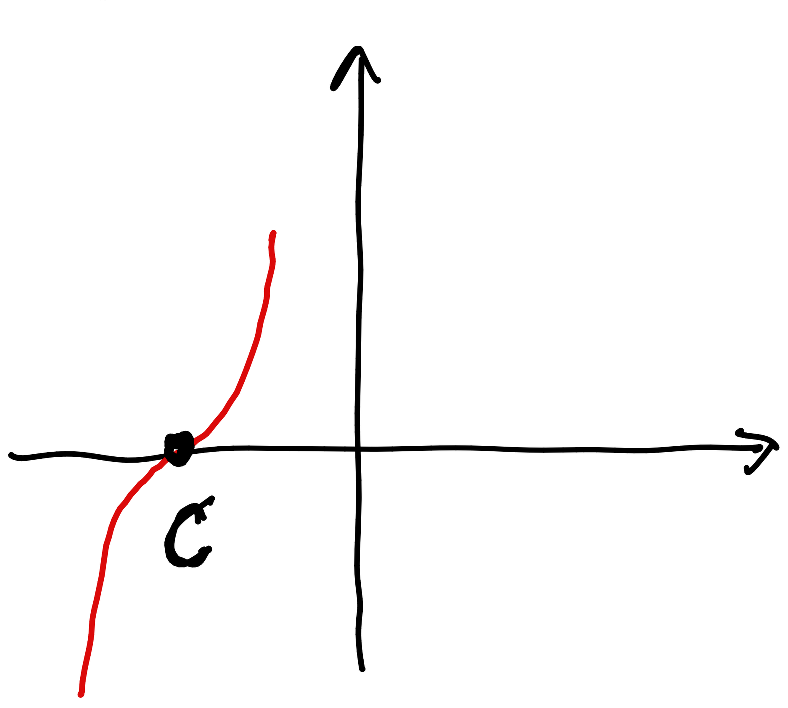
- If $k>1$ even, then the graph touches the x-axis at "c".



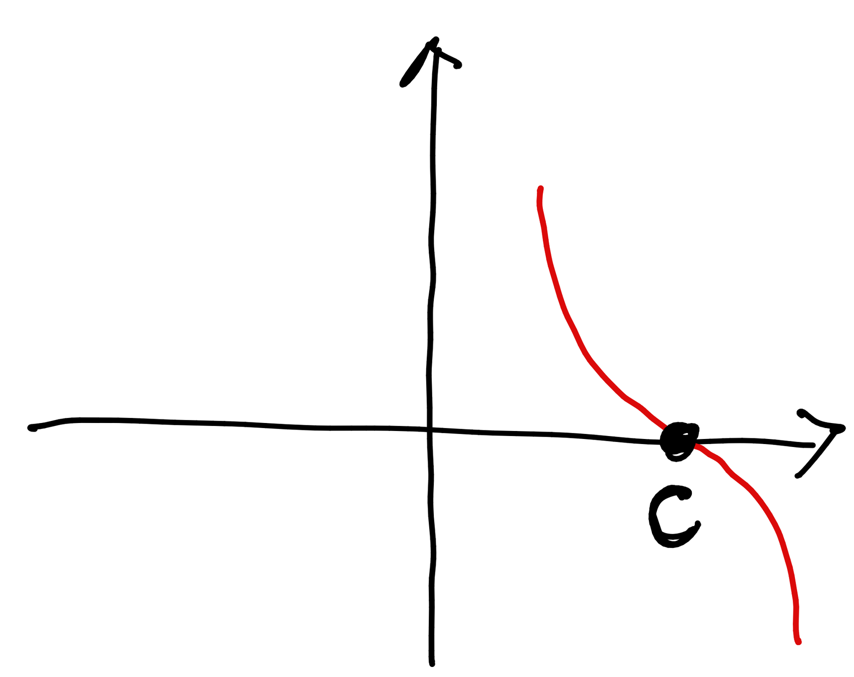
or



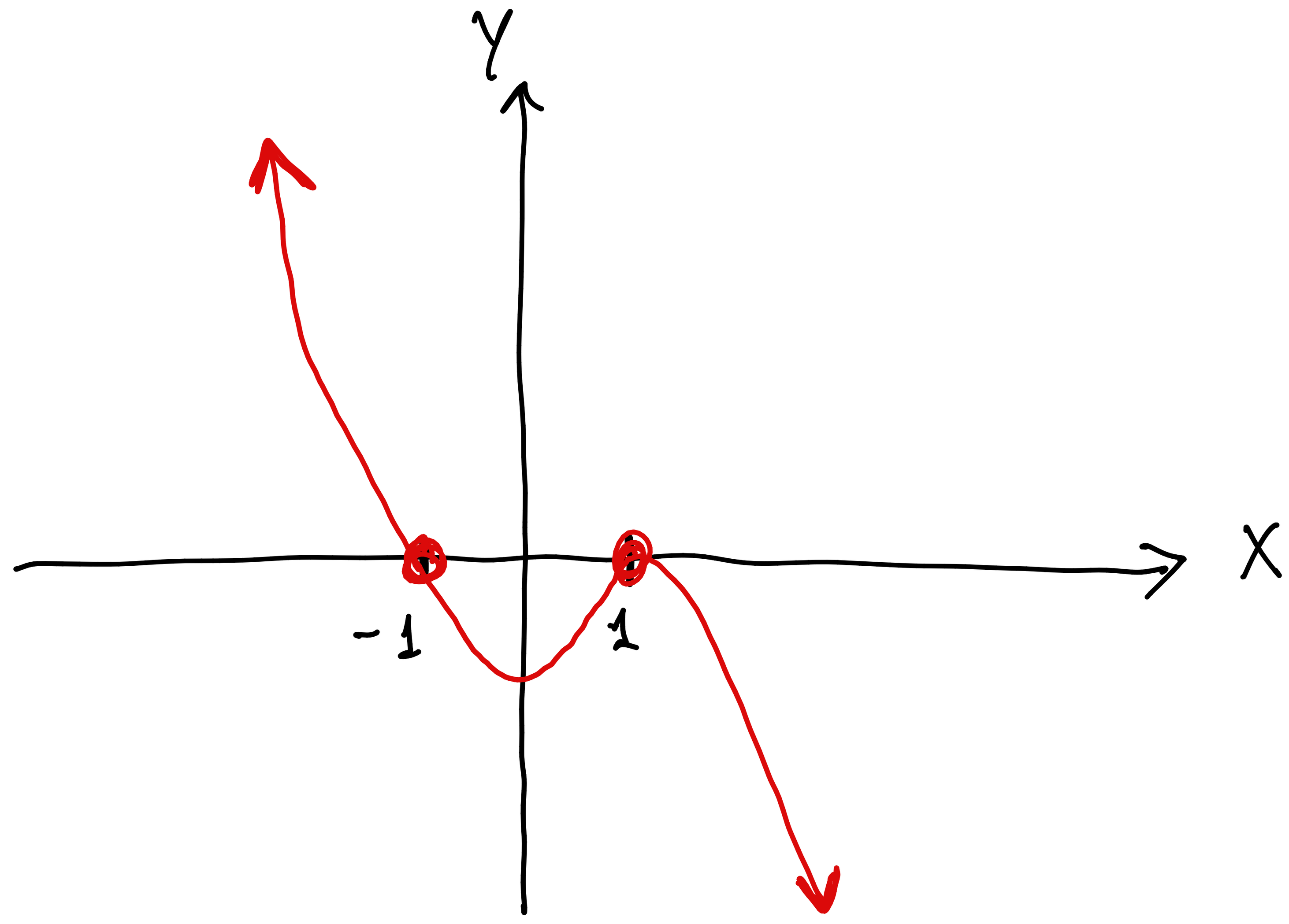
- If $k\geq1$ odd, then the graph crosses the x-axis at "c".



or



Ex 1: Let $f(x) = -3(x-1)^2(x+1)^1$.



$LC = -3 \cdot 1^2 \cdot 1 = -3$

$deg = (x^1)^2 \cdot (x^1)^1 = x^3 \rightarrow 3$

Factor $(x-1)^2$ ← mult

↑ root

→ $(x+1)^1$ ← mult

↑ -1 root

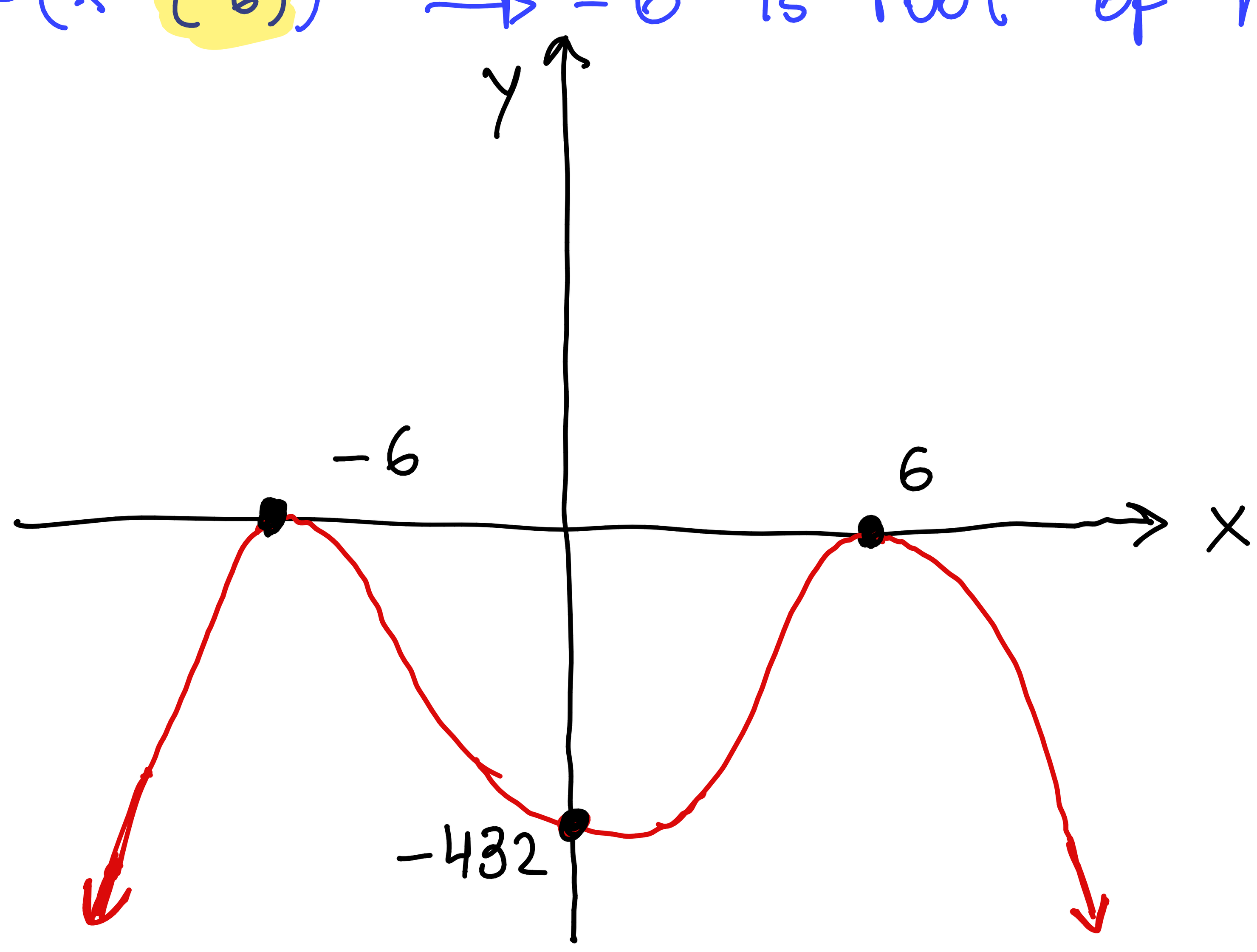
zeroes: -1 & 1

Ex 2: $g(x) = -\frac{1}{3}(x^2-36)^2$. Write $g(x) = -\frac{1}{3}\left((x-6)(x+6)\right)^2 = -\frac{1}{3}(x-6)^2(x+6)^2$.

$\boxed{LC = -\frac{1}{3}}$ & $\boxed{\text{degree} = 4}$

$(x-6)^2 \rightarrow$ 6 is a root of multiplicity 2.

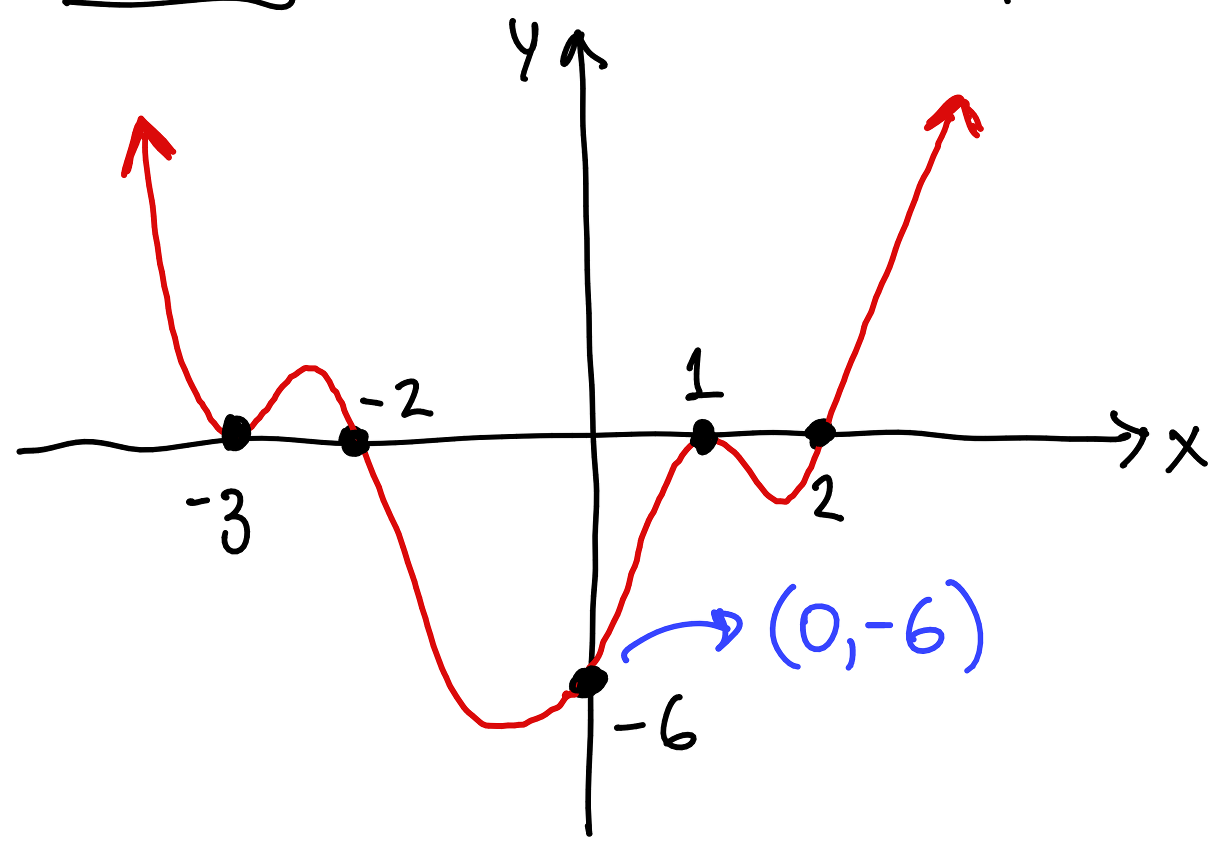
$(x+6)^2 = (x-(-6))^2 \rightarrow$ -6 is root of multiplicity 2.



$-\frac{1}{3}(x^2-36)^2$

$-\frac{1}{3} \cdot (-36)^2 = -432$

Ex 3: Find the polynomial from the given graph:



LC: graph ends up $\Rightarrow$ LC > 0

degree: same ends $\Rightarrow$ degree is even

roots: $(x+3)^{\text{even}} \cdot (x+2)^{\text{odd}} \cdot (x-1)^{\text{even}} \cdot (x-2)^{\text{odd}}$

zeroes: -3, -2, 1, 2

Suppose $f(x) = A(x+3)^2(x-1)^2(x+2)^1(x-2)^1 \rightarrow -6 = A\underbrace{(0+3)}^2(0-1)^2(0+2)^1(0-2)^1$

$\rightarrow -6 = A \cdot \underbrace{3^2}_{9} \cdot \underbrace{(-1)^2}_{1} \cdot \underbrace{2^1}_{2} \cdot \underbrace{(-2)^1}_{-2} \rightarrow \frac{-6}{-36} = \frac{-36A}{-36} \rightarrow \boxed{A = \frac{1}{6}}$

a) $\frac{1}{2}(x+3)^2(x-1)^2(x+2)(x-2)$

b) $\frac{1}{3}(x+3)^2(x-1)^2(x+2)(x-2)$

c) $\frac{1}{6}(x+3)^2(x-1)^2(x+2)(x-2)$

d) $\frac{1}{12}(x+3)^2(x-1)^2(x+2)(x-2)$